# 「離散数学・オートマトン」演習問題 13 (解答例)

2025/1/20

## 1 プッシュダウンオートマトン: Pushdown Automaton (PDA)

**課題 1** 式 (1.1) 及び図 1 で定義するプッシュダウンオートマトン $M$ を考える。
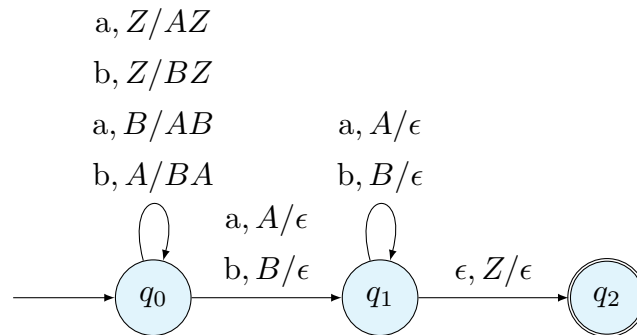Let us consider the pushdown automaton $M$ shown in Eq. (1.1) and Fig. 1.



図 1 PDA $M$

$$
\begin{aligned}
Q &= \{q_0. q_1, q_2\} \\
\Sigma &= \{\mathrm{a}, \mathrm{b}\} \\
\Gamma &= \{A, B, Z\} \\
F &= \{q_2\}
\end{aligned}
\tag{1.1}
$$

$$\delta\left(q_0, \mathrm{a}, Z\right) = \left(q_0, AZ\right), \qquad \delta\left(q_0, \mathrm{b}, Z\right) = \left(q_0, BZ\right),$$
$$\delta\left(q_0, \mathrm{a}, B\right) = \left(q_0, AB\right), \qquad \delta\left(q_0, \mathrm{b}, A\right) = \left(q_0, BA\right),$$
$$\delta\left(q_0, \mathrm{a}, A\right) = \left(q_1, \epsilon\right), \qquad \delta\left(q_0, \mathrm{b}, B\right) = \left(q_1, \epsilon\right),$$
$$\delta\left(q_1, \mathrm{b}, B\right) = \left(q_1, \epsilon\right), \qquad \delta\left(q_1, \mathrm{a}, A\right) = \left(q_1, \epsilon\right),$$
$$\delta\left(q_1, \epsilon, Z\right) = \left(q_2, \epsilon\right)$$

このとき、入力 ababbaba 及び babaabab に対する動作を示しなさい。

Show the behavior of the PDA $M$ for the input strings ababbaba and babaabab.

**解答例**

$$
\begin{aligned}
(q_0, \mathrm{ababbaba}, Z) &\vdash (q_0, \mathrm{babbaba}, AZ)\\
&\vdash (q_0, \mathrm{abbaba}, BAZ)\\
&\vdash (q_0, \mathrm{bbaba}, ABAZ)\\
&\vdash (q_0, \mathrm{baba}, BABAZ)\\
&\vdash (q_1, \mathrm{aba}, ABAZ)\\
&\vdash (q_1, \mathrm{ba}, BAZ)\\
&\vdash (q_1, \mathrm{a}, AZ)\\
&\vdash (q_1, \epsilon, Z)\\
&\vdash (q_2, \epsilon, \epsilon)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, \mathrm{babaabab}, Z) &\vdash (q_0, \mathrm{abaabab}, BZ)\\
&\vdash (q_0, \mathrm{baabab}, ABZ)\\
&\vdash (q_0, \mathrm{aabab}, BABZ)\\
&\vdash (q_0, \mathrm{abab}, ABABZ)\\
&\vdash (q_1, \mathrm{bab}, BABZ)\\
&\vdash (q_1, \mathrm{ab}, ABZ)\\
&\vdash (q_1, \mathrm{b}, BZ)\\
&\vdash (q_1, \epsilon, Z)\\
&\vdash (q_2, \epsilon, \epsilon)
\end{aligned}
$$

**課題 2**　式 (1.2) 及び図 2 で定義するプッシュダウンオートマトン $M$ を考える。

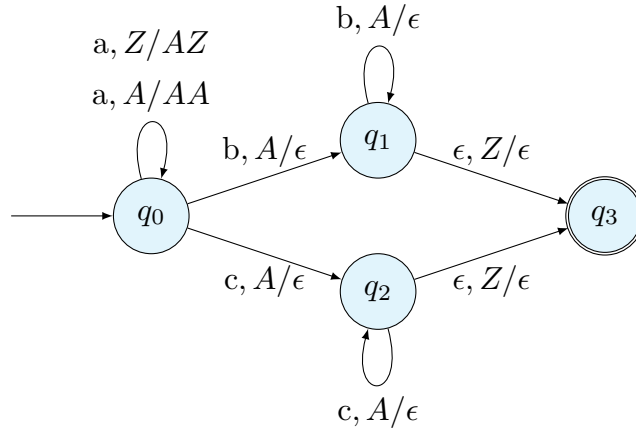Let us consider the pushdown automaton $M$ shown in Eq. (1.2) and Fig. 2.

図 2　PDA $M$

$$Q = \{q_0 \cdot q_1, q_2, q_3\}$$
$$\Sigma = \{\mathrm{a, b, c}\}$$
$$\Gamma = \{A, Z\}$$
$$F = \{q_3\}$$

(1.2)

$$\delta\left(q_0, \mathrm{a}, Z\right) = \left(q_0, AZ\right), \qquad \delta\left(q_0, \mathrm{a}, A\right) = \left(q_0, AA\right),$$
$$\delta\left(q_0, \mathrm{b}, A\right) = \left(q_1, \epsilon\right), \qquad \delta\left(q_0, \mathrm{c}, A\right) = \left(q_2, \epsilon\right),$$
$$\delta\left(q_1, \mathrm{b}, A\right) = \left(q_1, \epsilon\right), \qquad \delta\left(q_2, \mathrm{c}, A\right) = \left(q_2, \epsilon\right),$$
$$\delta\left(q_1, \epsilon, Z\right) = \left(q_3, \epsilon\right), \qquad \delta\left(q_2, \epsilon, Z\right) = \left(q_3, \epsilon\right)$$

このとき、入力 aaabbb 及び aacc に対する動作を示しなさい。

Show the behavior of the PDA $M$ for the input strings aaabbb and aacc.

**解答例**

$$
\begin{aligned}
(q_0, \mathrm{aaabbb}, Z) &\vdash (q_0, \mathrm{aabbb}, AZ) \\
&\vdash (q_0, \mathrm{abbb}, AAZ) \\
&\vdash (q_0, \mathrm{bbb}, AAAZ) \\
&\vdash (q_1, \mathrm{bb}, AAZ) \\
&\vdash (q_1, \mathrm{b}, AZ) \\
&\vdash (q_1, \epsilon, Z) \\
&\vdash (q_3, \epsilon, \epsilon)
\end{aligned}
$$

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$$(q_0, \text{aacc}, Z) \vdash (q_0, \text{acc}, AZ)$$
$$\vdash (q_0, \text{cc}, AAZ)$$
$$\vdash (q_2, \text{c}, AZ)$$
$$\vdash (q_2, \epsilon, Z)$$
$$\vdash (q_3, \epsilon, \epsilon)$$