「離散数学・オートマトン」演習問題 05 (解答例)

2024/11/7

1　グラフ: Graphs

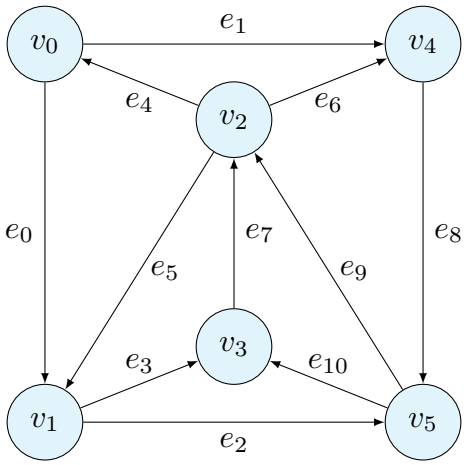
課題 1　以下のグラフ $G = (V, E)$ を図示しなさい。

Draw the following graph $G = (V, E)$.

$$V = \{v_0, v_1, v_2, v_3, v_4, v_5\} \tag{1.1}$$
$$E = \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\} \tag{1.2}$$

$$\begin{aligned}
\partial^+ e_0 &= v_0, & \partial^- e_0 &= v_1 & \partial^+ e_1 &= v_0, & \partial^- e_1 &= v_4 \\
\partial^+ e_2 &= v_1, & \partial^- e_2 &= v_5 & \partial^+ e_3 &= v_1, & \partial^- e_3 &= v_3 \\
\partial^+ e_4 &= v_2, & \partial^- e_4 &= v_0 & \partial^+ e_5 &= v_2, & \partial^- e_5 &= v_1 \\
\partial^+ e_6 &= v_2, & \partial^- e_6 &= v_4 & \partial^+ e_7 &= v_3, & \partial^- e_7 &= v_2 \\
\partial^+ e_8 &= v_4, & \partial^- e_8 &= v_5 & \partial^+ e_9 &= v_5, & \partial^- e_9 &= v_3 \\
\partial^+ e_{10} &= v_5, & \partial^- e_{10} &= v_3
\end{aligned}$$

解答例



Python のライブラリ `nexworkx` を使うことで、グラフを作図することができる。例は以下の Github から取得できる。

By using Python library `networkx`, we can draw the graph. The example can be obtained from the following Github.

`https://github.com/discrete-math-saga/Graph`

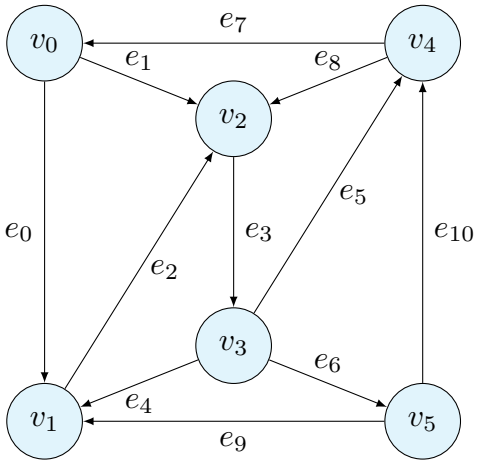
課題 2　以下のグラフ $G = (V, E)$ を図示しなさい。

Draw the following graph $G = (V, E)$.

$$V = \{v_0, v_1, v_2, v_3, v_4, v_5\} \tag{1.3}$$
$$E = \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\} \tag{1.4}$$

$$\begin{aligned}
\partial^+ e_0 &= v_0, & \partial^- e_0 &= v_1 & \partial^+ e_1 &= v_0, & \partial^- e_1 &= v_2 \\
\partial^+ e_2 &= v_1, & \partial^- e_2 &= v_2 & \partial^+ e_3 &= v_2, & \partial^- e_3 &= v_3 \\
\partial^+ e_4 &= v_3, & \partial^- e_4 &= v_1 & \partial^+ e_5 &= v_3, & \partial^- e_5 &= v4 \\
\partial^+ e_6 &= v_3, & \partial^- e_6 &= v_5 & \partial^+ e_7 &= v4, & \partial^- e_7 &= v_0 \\
\partial^+ e_8 &= v_4, & \partial^- e_8 &= v_2 & \partial^+ e_9 &= v_5, & \partial^- e_9 &= v_1 \\
\partial^+ e_{10} &= v_5, & \partial^- e_{10} &= v_4
\end{aligned}$$

解答例



課題 3　生徒の集合

$$P = \{\text{Bob}, \text{Ken}, \text{Mary}, \text{Ann}\} \tag{1.5}$$

と科目の集合

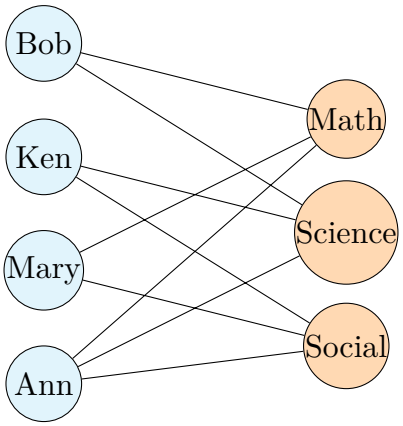
$$S = \{\text{Math}, \text{Science}, \text{Social}\} \tag{1.6}$$

を考える。関係 $R : P \to S$ は、「生徒 $p \in P$ は科目 $s \in S$ が得意である」を表すとする。以下の関係 R を図示せよ。

$$\begin{aligned} R = \{&(\text{Bob}, \text{Math}), (\text{Bob}, \text{Science}), (\text{Ken}, \text{Science}), (\text{Ken}, \text{Social}), \\ &(\text{Mary}, \text{Math}), (\text{Mary}, \text{Social}), (\text{Ann}, \text{Math}), (\text{Ann}, \text{Science}), (\text{Ann}, \text{Social})\} \end{aligned} \tag{1.7}$$

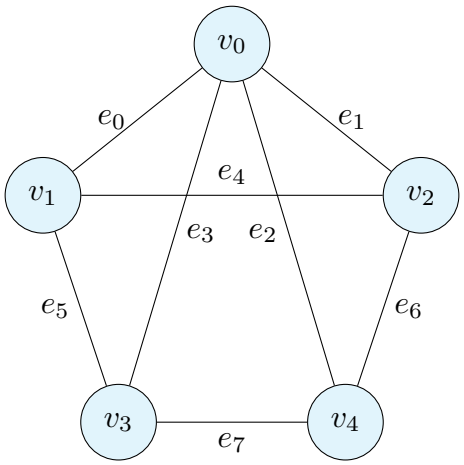
Consider the set of students in Eq. (1.5) and the set of subjects in Eq. (1.6). The relation $R : P \to S$ represents "student $p \in P$ is good at subject $s \in S$". Draw the relation R.

解答例



課題 4　以下のグラフに対して、Hamilton 閉路を列挙せよ。

Enumerate Hamilton cycles for the following graph.



解答例 以下の 8 種類の Hamilton 閉路がある。逆回りも含んでいる。

There are 8 Hamilton cycles as follows. The reverse direction is also included.

$$v_0, v_1, v_2, v_4, v_3$$
$$v_0, v_1, v_3, v_4, v_2$$
$$v_0, v_2, v_1, v_3, v_4$$
$$v_0, v_2, v_4, v_3, v_1$$
$$v_0, v_3, v_1, v_2, v_4$$
$$v_0, v_3, v_4, v_2, v_1$$
$$v_0, v_4, v_2, v_1, v_3$$
$$v_0, v_4, v_3, v_1, v_2$$

閉路を列挙する Python プログラムを示す。これも、前問同様の Github から取得できる。

The Python program to enumerate Hamilton cycles is shown below. This can also be obtained from the same Github as the previous question.

```
1  def enumerateHamilton(start:str, G:nx.Graph) -> list[list[str]]:
2      VHamilton = list()
3      VHamilton.append(start)
4      circuits = list()
5      enumerateHamiltonSub(start,start,VHamilton,G,circuits)
6      return circuits
7
8  def _enumerateHamiltonSub(currentNode:str, startNode:str, VHamilton:list[str],
   ↪  G:nx.Graph, circuits:list[list[str]]):
9      for edge in nx.edges(G,currentNode):
10         (f,t) = edge
11         if (t is startNode) and (len(G.nodes) == len(VHamilton)):
12             circuits.append(VHamilton)
13         else:
14             if t not in VHamilton:
15                 E = list(VHamilton)
16                 E.append(t)
17                 enumerateHamiltonSub(t,startNode,E,G,circuits)
```