有限オートマトン

離散数学・オートマトン
2024 年後期
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オートマトンと形式言語: Automata and Formal Languages

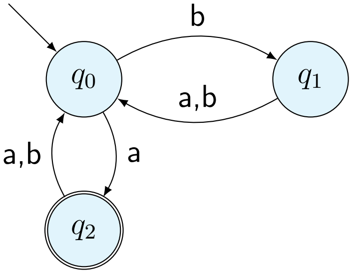
- オートマトン (Automaton)
  - 計算の抽象モデル: Abstract model of computation
  - テープからの入力による状態遷移: Transition based on input from tape
  - 「計算する」とは何かを考える: What does it mean to *compute*?
  - *automata* は複数形: *automata* is plural
- 形式言語 (Formal Language)
  - オートマトンが受理する言語: Language accepted by automaton
  - 文法を数学的に分析: Mathematical Analysis of grammars mathematically

決定性有限オートマトン
Deterministic Finite State Automata: DFA

$$M = \langle Q, \Sigma, \delta, q_0, F \rangle \tag{2.1}$$

- Q: **内部状態の有限集合**: Finite set of internal states
- Σ: **入力アルファベット、つまり入力記号の有限集合**: Finite set of input alphabets
- $\delta : Q \times \Sigma \to Q$: **状態遷移関数**: Transition function
    - $\delta(q, \mathrm{a}) = p$: **ある状態** q **で文字** a **を読むと、状態が** p **に遷移する**: At a state q, transition to a state p by reading character a
- $q_0 \in Q$: **初期状態**: Initial state
- $F \subseteq Q$: **受理状態の集合**: Set of accepting states
    - $q \in F$ **に到達する入力を受理する**: Accept inputs that reaches $q \in F$

例 2.1:



$$Q = \{q_0, q_1, q_2\}$$
$$\Sigma = \{\mathrm{a}, \mathrm{b}\}$$
$$F = \{q_2\}$$

遷移関数 (transition function) δ

	a	b
q_0	q_2	q_1
q_1	q_0	q_0
q_2	q_0	q_0

動作イメージ: Images of Operation

テープヘッドが移動して、テープ上の文字を読み取る。: Reading an alphabet on the tape and moving the tape head.

a	b	a	b	b	a	a
q_0						

$$(q_0, \text{ababbaa}) \vdash_M (q_2, \text{babbaa})$$

a	b	a	b	b	a	a
	q_2					

$$(q_2, \text{babbaa}) \vdash_M (q_0, \text{abbaa})$$

a	b	a	b	b	a	a
		q_0				

$$
\begin{aligned}
(q_0, \mathsf{ababbaa}) &\vdash_M (q_2, \mathsf{babbaa}) \\
&\vdash_M (q_0, \mathsf{abbaa}) \\
&\vdash_M (q_2, \mathsf{bbaa}) \\
&\vdash_M (q_0, \mathsf{baa}) \\
&\vdash_M (q_1, \mathsf{aa}) \\
&\vdash_M (q_0, \mathsf{a}) \\
&\vdash_M (q_2, \epsilon)
\end{aligned}
$$

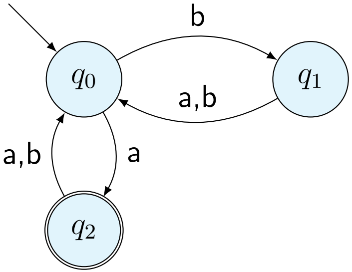
遷移関数 δ

	a	b
q_0	q_2	q_1
q_1	q_0	q_0
q_2	q_0	q_0

ϵは長さ 0 の文字列を表す

ϵ represents a string of length 0

例:2.1 への入力 bbaba



$\vdash_M$ の推移的閉包と受理言語
Transitive Closure of $\vdash_M$ and Accepted Languages

- 入力 $w \in \Sigma^*$(Σ^* は Σ の要素の 0 個以上の列) によって、初期状態 q_0 から状態 q へ遷移し、テープに残っている文字列が w' であるとき
When the input $w \in \Sigma^*$ (Σ^* is a sequence of 0 or more elements of Σ) causes a transition from the initial state q_0 to the state q, and the remaining string on the tape is w'

$$(q_0, w) \vdash_M^* (q, w') \tag{3.1}$$

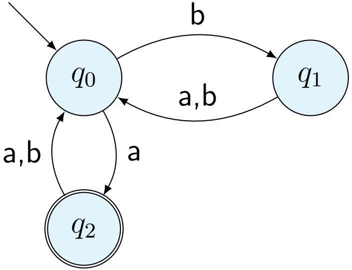
- M が入力 w を受理: M accepts the input w

$$(q_0, w) \vdash_M^* (q_F, \epsilon), \quad q_F \in F \tag{3.2}$$

- 受理言語: Accepted Language

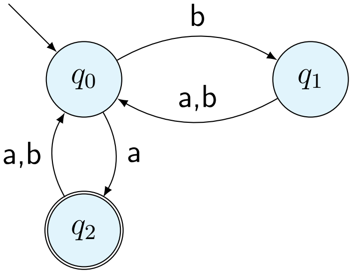
$$L(M) = \{w \in \Sigma^* \mid (q_0, w) \vdash_M^* (q_F, \epsilon), \quad q_F \in F\} \tag{3.3}$$

例:2.1 の場合



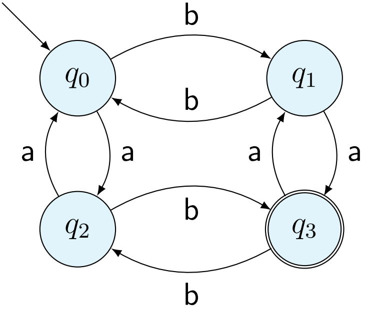
$$
\begin{aligned}
(q_0, \mathsf{aaaba}) &\vdash (q_2, \mathsf{aaba}) \vdash (q_0, \mathsf{aba}) \\
&\vdash (q_2, \mathsf{ba}) \vdash (q_0, \mathsf{a}) \vdash (q_2, \epsilon) \\
(q_0, \mathsf{babaa}) &\vdash (q_1, \mathsf{abaa}) \vdash (q_0, \mathsf{baa}) \\
&\vdash (q_1, \mathsf{aa}) \vdash (q_0, \mathsf{a}) \vdash (q_2, \epsilon)
\end{aligned}
$$

受理する入力の例
Example of Accepted Inputs



`a, aaa, aba, baa, bba, aaaaa, aaaba, abaaa, babaa, babba, bbbaa, bbbba`

例 3.1:

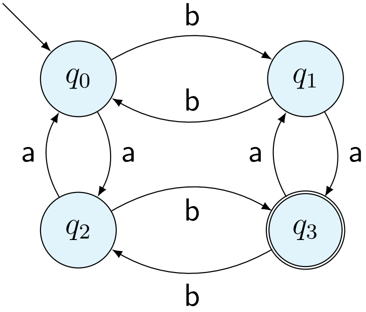


$$Q = \{q_0, q_1, q_2, q_3\}$$
$$\Sigma = \{\mathsf{a}, \mathsf{b}\}$$
$$F = \{q_3\}$$

遷移関数 δ

	a	b
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

例 3.1: 動作例: Example of Operation

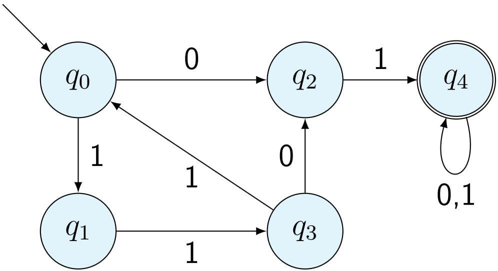


$$
\begin{aligned}
(q_0, \mathsf{aaaaab}) &\vdash (q_2, \mathsf{aaaab}) \vdash (q_0, \mathsf{aaab}) \vdash (q_2, \mathsf{aab}) \\
&\vdash (q_0, \mathsf{ab}) \vdash (q_2, \mathsf{b}) \vdash (q_3, \epsilon) \\
(q_0, \mathsf{abbaba}) &\vdash (q_2, \mathsf{bbaba}) \vdash (q_3, \mathsf{baba}) \vdash (q_2, \mathsf{aba}) \\
&\vdash (q_0, \mathsf{ba}) \vdash (q_1, \mathsf{a}) \vdash (q_3, \epsilon)
\end{aligned}
$$

例 3.1: 受理する文字列例 (長さ 5 まで) Example of Accepted Strings (up to length 5)

ab ba aaab aaba abaa abbb baaa babb bbab bbba

例 3.2:



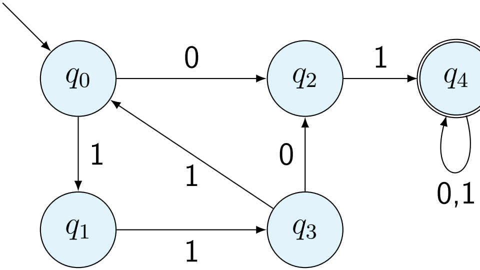
$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$
$$\Sigma = \{0, 1\}$$
$$F = \{q_4\}$$

遷移関数 δ

	0	1
q_0	q_2	q_1
q_1		q_3
q_2		q_4
q_3	q_2	q_0
q_4	q_4	q_4

空欄に注意

例 3.2: 動作例: Example of Operation



$$
\begin{aligned}
(q_0, 1110101) &\vdash (q_1, 110101) \vdash (q_3, 10101) \vdash (q_0, 0101) \\
&\vdash (q_2, 101) \vdash (q_4, 01) \vdash (q_4, 1) \vdash (q_4, \epsilon) \\
(q_0, 1101010) &\vdash (q_1, 101010) \vdash (q_3, 01010) \vdash (q_2, 1010) \\
&\vdash (q_4, 010) \vdash (q_4, 10) \vdash (q_4, 0) \vdash (q_4, \epsilon)
\end{aligned}
$$

例 3.2: 受理する文字列例 (長さ 5 まで)
Example of Accepted Strings (up to length 5)

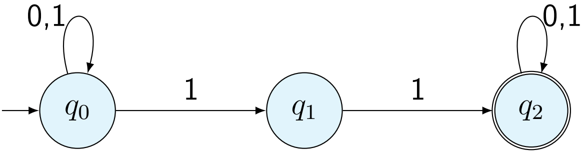
01, 010, 011, 0100, 0101, 0110, 0111, 1101, 01000, 01001, 01010, 01011, 01100, 01101, 01110, 01111, 11010, 11011, 11101

非決定性有限オートマトン
Non-deterministic Finite State Automata: NFA

$$M = \langle Q, \Sigma, \delta, q_0, F \rangle \tag{4.1}$$

- Q: **内部状態の集合**: Finite set of internal states
- Σ: **入力アルファベット**: Input alphabet
- $\delta : Q \times \Sigma \to 2^Q$: **状態遷移関数**: Transition function
  - 2^Q は、Q のべき集合、つまり Q の部分集合の族。遷移先が複数であることを許容することに注意。: 2^Q is the power set of Q. Note that it allows multiple transition possibilities.
- $q_0 \in Q$: **初期状態**: Initial state
- $F \subseteq Q$: **受理状態**: Accepting states

例 4.1:

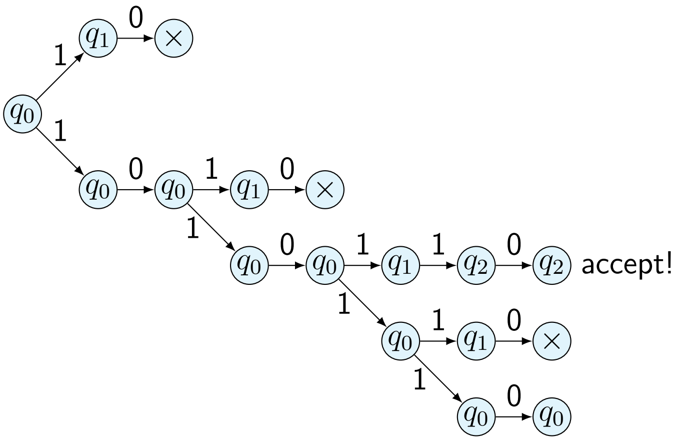


$$Q = \{q_0, q_1, q_2\}, \quad \Sigma = \{0, 1\}, \quad F = \{q_2\}$$

遷移関数 δ

	0	1
q_0	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\emptyset$	$\{q_2\}$
q_2	$\{q_2\}$	$\{q_2\}$

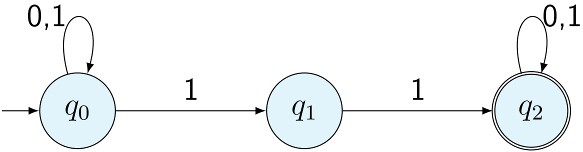
動作例: 入力 1010110



受理条件: Accept conditions

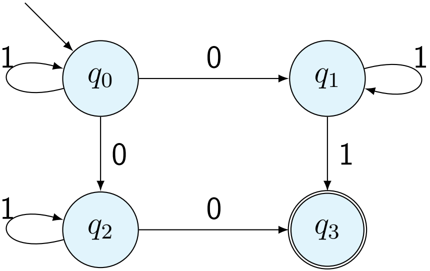
- 入力が引き起こす状態遷移のうちで、受理状態に至る場合が一つでもあれば、その入力を受理する。
  If there is at least one state transition caused by the input that reaches an accepting state, the input is accepted.

長さ 5 以下の受理入力



11, 011, 110, 111, 0011, 0110, 0111, 1011, 1100, 1110, 1111, 00011, 00110, 00111, 01011, 01100, 01110, 01111, 10011, 10110, 10111, 11000, 11011, 11100, 11110, 11111

例 4.2:



$$Q = \{q_0, q_1, q_2, q_2\}, \quad \Sigma = \{0, 1\}, \quad F = \{q_3\}$$

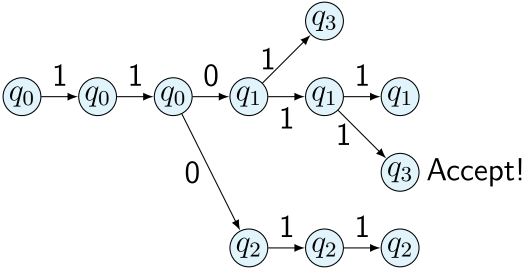
遷移関数 δ

δ	0	1
q_0	$\{q_1, q_2\}$	$\{q_0\}$
q_1	$\emptyset$	$\{q_1, q_3\}$
q_2	$\{q_3\}$	$\{q_2\}$
q_3	$\emptyset$	$\emptyset$

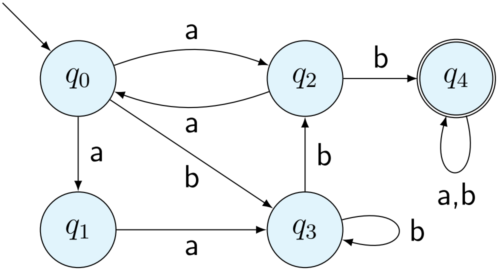
長さ 5 以下の受理入力

00, 01, 010, 011, 100, 101, 0110, 0111, 1010, 1011, 1100, 1101, 01110, 01111, 10110, 10111, 11010, 11011, 11100, 11101

動作例: 入力 11011



例 4.3:

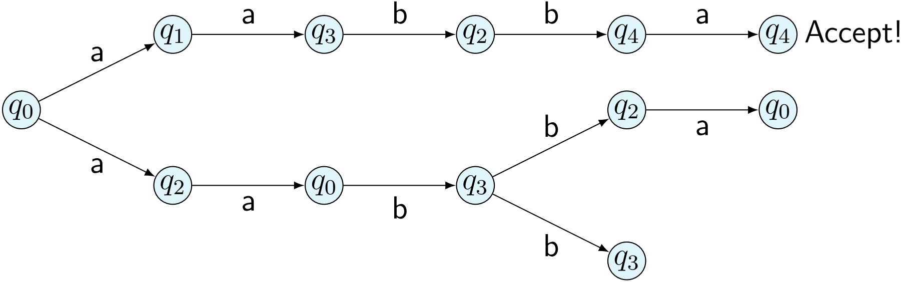


$$Q = \{q_0, q_1, q_2, q_3, q_4\}, \Sigma = \{\mathsf{a}, \mathsf{b}\}, F = \{q_4\}$$

遷移関数 δ

	a	b
q_0	$\{q_1, q_2\}$	$\{q_3\}$
q_1	$\{q_3\}$	$\emptyset$
q_2	$\emptyset$	$\{q_4\}$
q_3	$\emptyset$	$\{q_2, q_3\}$
q_4	$\{q_4\}$	$\{q_4\}$

動作例: 入力 aabba



疑問: Questions

- **オートマトンが受理する文字列の集合を記述する方法**: How to describe the set of strings accepted by an automaton
  - **文字列パターンを記述する方法**: How to describe string patterns
- NFA **と** DFA **は本質的に異なるのか**: Are NFA and DFA fundamentally different?
  - **受理する文字列集合は異なるのか**: Are the sets of accepted strings different?
  - **能力は異なるか**: Are their capabilities different?